

INTEGRALINIS SKAIČIAVIMAS

1. Tiesioginis integravimas

1. $\int (2x+5)dx$; Ats.: $x^2 + 5x + C$.
2. $\int (4x^3 + 3x^2 - 3)dx$; Ats.: $x^4 + x^3 - 3x + C$.
3. $\int (12x^3 - 6x + 4)dx$; Ats.: $3x^4 - 3x^2 + 4x + C$.
4. $\int (x-1)(x+2)dx$; Ats.: $\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + C$.
5. $\int (3x^2 - 6x + 2)dx$; Ats.: $x^3 - 3x^2 + 2x + C$.
6. $\int (5x^4 - 6x^2 - 1)dx$; Ats.: $x^5 - 2x^3 - x + C$.
7. $\int (1 + 5\cos x - 3x^2)dx$; Ats.: $x + 5\sin x - x^3 + C$.
8. $\int \left(4e^x - 3x^2 + \frac{1}{x^2}\right)dx$; Ats.: $4e^x - x^3 - \frac{1}{x} + C$.
9. $\int \left(3^x + \frac{2}{x} - e^x\right)dx$; Ats.: $\frac{3^x}{\ln 3} + 2\ln|x| - e^x + C$.
10. $\int \left(\frac{1}{\cos^2 x} - \frac{1}{2x} + 4\right)dx$; Ats.: $\tan x - \frac{1}{2}\ln|x| + 4x + C$.
11. $\int \left(x^2 - \frac{1}{x^2}\right)dx$; Ats.: $\frac{x^3}{3} + \frac{1}{x} + C$.
12. $\int (\sin x - 2x)dx$; Ats.: $-\cos x - x^2 + C$.
13. $\int \left(4x^3 + \frac{3}{x}\right)^2 dx$; Ats.: $\frac{16}{7}x^7 + 8x^3 - \frac{9}{x} + C$.
14. $\int \frac{4x^4 - 2x^2 + 3}{x} dx$; Ats.: $x^4 - x^2 + 3\ln|x| + C$.
15. $\int (\sqrt[3]{x^2} - \sqrt{x})dx$; Ats.: $\frac{3}{5}x\sqrt[3]{x^2} - \frac{2}{3}x\sqrt{x} + C$.
16. $\int x\left(\frac{2}{x} + \frac{x}{2}\right)dx$; Ats.: $2x + \frac{1}{6}x^3 + C$.
17. $\int \frac{3x^2\sqrt[6]{x}\sqrt{x}}{\sqrt[3]{x^2}} dx$; Ats.: $x^3 + C$.
18. $\int \frac{(x^2+1)^2}{x^3} dx$; Ats.: $\frac{1}{2}x^2 + 2\ln|x| - \frac{1}{2x^2} + C$.
19. $\int \frac{(x+1)^2}{\sqrt{x}} dx$; Ats.: $\frac{2}{5}x^2\sqrt{x} + \frac{4}{3}x\sqrt{x} + 2\sqrt{x} + C$.
20. $\int e^x\left(1 - \frac{e^{-x}}{x^2}\right)dx$; Ats.: $e^x + \frac{1}{x} + C$.
21. $\int \frac{x^3 + 2x}{x\sqrt{x}} dx$; Ats.: $\frac{2}{5}x^2\sqrt{x} + 4\sqrt{x} + C$.
22. $\int 2^x \cdot 3^x dx$; Ats.: $\frac{6^x}{\ln 6} + C$.
23. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$; Ats.: $-ctgx - tgx + C$.
24. $\int (\sin x + 3\cos x)dx$; Ats.: $-\cos x + 3\sin x + C$.
25. $\int \frac{3 - 2ctg^2 x}{\cos^2 x} dx$; Ats.: $3tgx + 2ctgx + C$.
26. $\int \left(\frac{2}{1+x^2} - \frac{3}{\sqrt{1-x^2}}\right)dx$; Ats.: $2\arctgx - 3\arcsin x + C$.

2. Integravimas keičiant kintamuosius

1. $\int x^2 \sqrt{1+x^3} dx$; Ats.: $\frac{2}{9}\sqrt{(1+x^3)^3} + C$.
2. $\int \frac{3x dx}{\sqrt{5-2x^2}}$; Ats.: $-\frac{3}{2}\sqrt{5-2x^2} + C$.
3. $\int \frac{6z^3 dz}{5z^4 - 2}$; Ats.: $\frac{3}{10}\ln|5z^4 - 2| + C$.
4. $\int \frac{z^2 dz}{5z^3 + 1}$; Ats.: $\frac{1}{15}\ln|5z^3 + 1| + C$.
5. $\int e^{3x+1} dx$; Ats.: $\frac{1}{3}e^{3x+1} + C$.
6. $\int e^{5x^2-1} 2x dx$; Ats.: $\frac{1}{5}e^{5x^2-1} + C$.
7. $\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}} dx$; Ats.: $\frac{2}{3}e^{\sqrt{3x-1}} + C$.
8. $\int e^{3x^2+6x+5} (x+1) dx$; Ats.: $\frac{1}{6}e^{3x^2+6x+5} + C$.

$$9. \int 5^{4x^2-3} 8x dx; \quad \text{Ats.: } \frac{5^{4x^2-3}}{\ln 5} + C.$$

$$11. \int \frac{8^{tgx}}{5 \cos^2 x} dx; \quad \text{Ats.: } \frac{8^{tgx}}{5 \ln 8} + C.$$

$$13. \int \cos^2 t dt; \quad \text{Ats.: } \frac{1}{2}t + \frac{1}{4} \sin 2t + C.$$

$$15. \int \sin^3 x \cos x dx; \quad \text{Ats.: } \frac{1}{4} \sin^4 x + C.$$

$$17. \int \frac{xdx}{\sin^2\left(\frac{x^2}{5}\right)}; \quad \text{Ats.: } -\frac{5}{2} \operatorname{ctg}\left(\frac{x^2}{5}\right) + C.$$

$$19. \int \frac{4dx}{\sqrt{1-16x^2}}; \quad \text{Ats.: } \arcsin 4x + C.$$

$$21. \int \frac{dx}{\sqrt{1-(x+2)^2}}; \quad \text{Ats.: } \arcsin(x+2) + C.$$

$$23. \int \frac{5dx}{3+12x^2}; \quad \text{Ats.: } \frac{5}{6} \operatorname{arctg} 2x + C.$$

$$25. \int \frac{\sqrt{\ln x}}{x} dx; \quad \text{Ats.: } \frac{2}{3} \sqrt{(\ln x)^3} + C.$$

$$27. \int \cos x \sin 3x dx; \quad \text{Ats.: } -\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x + C.$$

$$29. \int \frac{\sin^3 x dx}{\sqrt{\cos x}}; \quad \text{Ats.: } 2\sqrt{\cos x} \left(\frac{\cos^2 x}{5} - 1 \right) + C.$$

$$10. \int \frac{6^{\frac{1}{x^2}} dx}{2x^3}; \quad \text{Ats.: } \frac{-6^{\frac{1}{x^2}}}{4 \ln 6} + C.$$

$$12. \int \cos(5t^2-3) 2t dt; \quad \text{Ats.: } \frac{1}{5} \sin(5t^2-3) + C.$$

$$14. \int \sin^2 x dx; \quad \text{Ats.: } \frac{1}{2}x - \frac{1}{4} \sin 2x + C.$$

$$16. \int \frac{5x^2 dx}{\sin^2(8x^3-2)}; \quad \text{Ats.: } -\frac{5}{24} \operatorname{ctg}(8x^3-2) + C.$$

$$18. \int \frac{6dx}{\sqrt{9-36x^2}}; \quad \text{Ats.: } \arcsin 2x + C.$$

$$20. \int \frac{dx}{\sqrt{5-3x^2}}; \quad \text{Ats.: } \frac{\sqrt{3}}{3} \arcsin\left(\sqrt{\frac{3}{5}}x\right) + C.$$

$$22. \int \frac{dx}{\sqrt{4-(x-3)^2}}; \quad \text{Ats.: } \arcsin \frac{x-3}{2} + C.$$

$$24. \int \frac{4du}{5+6u^2}; \quad \text{Ats.: } \frac{2\sqrt{30}}{15} \operatorname{arctg} \frac{\sqrt{30}z}{5} + C.$$

$$26. \int \frac{x^3 dx}{\sqrt{1-x^8}}; \quad \text{Ats.: } \frac{1}{4} \arcsin x^4 + C.$$

$$28. \int \cos^3 x dx; \quad \text{Ats.: } \sin x - \frac{1}{3} \sin^3 x + C.$$

$$30. \int \frac{\cos x}{1+2 \sin x} dx; \quad \text{Ats.: } \frac{1}{2} \ln|1+2 \sin x| + C.$$

3. Integravimas dalimis

$$1. \int x \sin x dx; \quad \text{Ats.: } -x \cos x + \sin x + C.$$

$$3. \int x^2 e^x dx; \quad \text{Ats.: } x^2 e^x - 2x e^x + 2e^x + C.$$

$$5. \int x e^{-2x} dx; \quad \text{Ats.: } -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C.$$

$$7. \int (1-x) \sin x dx; \quad \text{Ats.: } (x-1) \cos x - \sin x + C.$$

$$9. \int x \sin 2x dx; \quad \text{Ats.: } -\frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + C.$$

$$11. \int e^{2x} \cos x dx; \quad \text{Ats.: } \frac{1}{5} e^{2x} (\sin x + 2 \cos x) + C.$$

$$13. \int x \operatorname{arctg} x dx; \quad \text{Ats.: } \frac{1}{2} (x^2 \operatorname{arctg} x + \operatorname{arctg} x - x) + C.$$

$$15. \int \frac{xdx}{\sin^2 x}; \quad \text{Ats.: } \ln|\sin x| - x \operatorname{ctg} x + C.$$

$$17. \int \arccos x dx; \quad \text{Ats.: } x \arccos x + \sqrt{1-x^2} + C.$$

$$19. \int x^3 e^x dx; \quad \text{Ats.: } e^x (x^3 - 3x^2 + 6x - 6) + C.$$

$$2. \int \ln x dx; \quad \text{Ats.: } x \ln x - x + C.$$

$$4. \int x e^x dx; \quad \text{Ats.: } e^x (x-1) + C.$$

$$6. \int (x-1) e^{2x} dx; \quad \text{Ats.: } \frac{1}{2} (x-1) e^{2x} - \frac{1}{4} e^{2x} + C.$$

$$8. \int x \cos x dx; \quad \text{Ats.: } x \sin x + \cos x + C.$$

$$10. \int e^x \sin x dx; \quad \text{Ats.: } \frac{1}{2} e^x (\sin x - \cos x) + C.$$

$$12. \int \operatorname{arctg} x dx; \quad \text{Ats.: } x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C.$$

$$14. \int \frac{\ln x}{x^2} dx; \quad \text{Ats.: } -\frac{\ln x}{x} - \frac{1}{x} + C.$$

$$16. \int \frac{xdx}{\cos^2 x}; \quad \text{Ats.: } x \operatorname{tg} x + \ln|\cos x| + C.$$

$$18. \int x \cdot 3^x dx; \quad \text{Ats.: } \frac{3^x}{\ln^2 3} (x \ln 3 - 1) + C.$$

$$20. \int \ln^2 x dx; \quad \text{Ats.: } x(\ln^2 x - 2 \ln x + 2) + C.$$

4. Racionaliųjų trupmenų integravimas

1. $\int \frac{x}{x+4} dx$; Ats.: $x - 4 \ln|x+4| + C$.
2. $\int \frac{(1+x)^2}{x^2+1} dx$; Ats.: $x + \ln(x^2+1) + C$.
3. $\int \frac{x^4 dx}{x^2+1}$; Ats.: $\frac{x^3}{3} - x + \arctg x + C$.
4. $\int \frac{dx}{x(x-1)}$; Ats.: $\ln \left| \frac{x-1}{x} \right| + C$.
5. $\int \frac{dx}{(x+1)(2x-3)}$; Ats.: $\frac{1}{5} \ln \left| \frac{2x-3}{x+1} \right| + C$.
6. $\int \frac{dx}{x^2+3x-10}$; Ats.: $\frac{1}{7} \ln \left| \frac{x-2}{x+5} \right| + C$.
7. $\int \frac{2x+7}{x^2+x-2} dx$; Ats.: $\ln \left| \frac{(x-1)^3}{x+2} \right| + C$.
8. $\int \frac{6x^2-13x+4}{x^3-3x^2+2x} dx$; Ats.: $\ln|x^2(x-1)^3(x-2)| + C$.
9. $\int \frac{x^3}{x-2} dx$; Ats.: $\frac{x^3}{3} + x^2 + 4x + 8 \ln|x-2| + C$.
10. $\int \frac{6x-4}{x^3-4x} dx$; Ats.: $\ln \left| \frac{x(x-2)}{(x+2)^2} \right| + C$.
11. $\int \frac{x-4}{(x-2)(x-3)} dx$; Ats.: $\ln \left| \frac{(x-2)^2}{x-3} \right| + C$.
12. $\int \frac{xdx}{(x+1)(2x+1)}$; Ats.: $\ln \frac{|x+1|}{\sqrt{2x+1}} + C$.
13. $\int \frac{3x^2+2x-3}{x^3-x} dx$; Ats.: $\ln \left| \frac{x^3(x-1)}{x+1} \right| + C$.
14. $\int \frac{(x+1)^3}{x^2-x} dx$; Ats.: $\frac{x^2}{2} + 4x + \ln \left| \frac{(x-1)^8}{x} \right| + C$.
15. $\int \frac{x+2}{x^3-2x^2} dx$; Ats.: $\frac{1}{x} + \ln \left| \frac{x-2}{x} \right| + C$.
16. $\int \frac{x^2+2x+2}{x^3+x} dx$; Ats.: $\ln \frac{x^2}{\sqrt{x^2+1}} + 2 \arctg x + C$.

5. Kai kurių trigonometrinių funkcijų integravimas

1. $\int \sin^3 x dx$; Ats.: $-\cos x + \frac{\cos^3 x}{3} + C$.
2. $\int \cos^3 x dx$; Ats.: $\sin x - \frac{1}{3} \sin^3 x + C$.
3. $\int \sin 5x \sin 3x dx$; Ats.: $\frac{1}{4} \sin 2x - \frac{1}{16} \sin 8x + C$.
4. $\int \cos^4 x dx$; Ats.: $\frac{3}{8} x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$.
5. $\int \cos 4x \cos x dx$; Ats.: $\frac{1}{6} \sin 3x + \frac{1}{10} \sin 5x + C$.
6. $\int \sin 7x \cos 3x dx$; Ats.: $-\frac{1}{8} \cos 4x - \frac{1}{20} \cos 10x + C$.
7. $\int (1+2\cos x)^2 dx$; Ats.: $3x + 4 \sin x + \sin 2x + C$.
8. $\int (1-\sin 2x)^2 dx$; Ats.: $\frac{3x}{2} + \cos 2x - \frac{\sin 4x}{8} + C$.
9. $\int \sin^2 x \cos^2 x dx$; Ats.: $\frac{x}{8} - \frac{\sin 4x}{32} + C$.
10. $\int \sin^4 x \cos^4 x dx$; Ats.: $\frac{3x}{128} - \frac{\sin 4x}{128} + \frac{\sin 8x}{1024} + C$.
11. $\int \frac{dx}{\sin x}$; Ats.: $\ln \left| \operatorname{tg} \frac{x}{2} \right| + C$.
12. $\int \sin^2 x \cos x dx$; Ats.: $\frac{\sin^3 x}{3} + C$.

6. Kai kurių iracionaliųjų funkcijų integravimas

1. $\int \frac{\sqrt{x} dx}{x+1}$; Ats.: $2(\sqrt{x} - \arctg \sqrt{x}) + C$.
2. $\int \frac{x^3 dx}{\sqrt{4-x^2}}$; Ats.: $-\frac{(x^2+8)\sqrt{4-x^2}}{3} + C$.
3. $\int x\sqrt{x+2} dx$; Ats.: $\frac{2}{5} \sqrt{(x+5)^5} - \frac{4}{3} \sqrt{(x+2)^3} + C$.
4. $\int \frac{dx}{(1+\sqrt[4]{x})\sqrt{x}}$; Ats.: $4\sqrt[4]{x} - 4 \ln(\sqrt[4]{x}+1) + C$.
5. $\int \frac{dx}{\sqrt{x}(1+\sqrt[3]{x})}$; Ats.: $6\sqrt[6]{x} - 6 \arctg \sqrt[6]{x} + C$.
6. $\int \frac{dx}{\sqrt{2x+5}}$; Ats.: $\sqrt{2x} - 5 \ln(\sqrt{2x}+5) + C$.
7. $\int \frac{\sqrt{x}}{1+\sqrt{x}} dx$; Ats.: $x - 2\sqrt{x} + 2 \ln(1+\sqrt{x}) + C$.
8. $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$; Ats.: $2 \ln(1+\sqrt{x}) + C$.

$$9. \int \frac{x dx}{\sqrt{2x+1}+1}; \text{ Ats.: } \frac{2x+1}{12} (2\sqrt{2x+1}-3) + C.$$

$$10. \int \frac{dx}{\sqrt{x}-1}; \text{ Ats.: } 2\sqrt{x} + 2\ln|\sqrt{x}-1| + C.$$

7. Įvairių funkcijų integravimas

$$1. \int \frac{dx}{\sqrt[3]{x}}; \text{ Ats.: } \frac{3}{2}\sqrt[3]{x} + C.$$

$$2. \int \left(x\sqrt{x} - \frac{1}{x^2} \right) dx; \text{ Ats.: } \frac{2}{5}x^2\sqrt{x} + \frac{1}{x} + C.$$

$$3. \int \left(x + \frac{1}{x} \right)^2 dx; \text{ Ats.: } \frac{1}{3}x^3 + 2x - \frac{1}{x} + C.$$

$$4. \int \sqrt{3-2x} dx; \text{ Ats.: } -\frac{1}{3}\sqrt{(3-2x)^3} + C.$$

$$5. \int \frac{dx}{6-5x}; \text{ Ats.: } -\frac{1}{5}\ln|6-5x| + C.$$

$$6. \int \frac{x dx}{x^2-1}; \text{ Ats.: } \frac{1}{2}\ln|x^2-1| + C.$$

$$7. \int \cos x (\sin^5 x + 1) dx; \text{ Ats.: } \frac{1}{6}\sin^6 x + \sin x + C.$$

$$8. \int x\sqrt{x^2-4} dx; \text{ Ats.: } \frac{1}{3}(x^2-4)^{\frac{3}{2}} + C.$$

$$9. \int \frac{dx}{x^2+4}; \text{ Ats.: } \frac{1}{2}\arctg \frac{x}{2} + C.$$

$$10. \int \frac{dx}{\sqrt{4-x^2}}; \text{ Ats.: } \arcsin \frac{x}{2} + C.$$

$$11. \int \sin 5x dx; \text{ Ats.: } -\frac{1}{5}\cos 5x + C.$$

$$12. \int \cos^3 x \sin x dx; \text{ Ats.: } -\frac{1}{4}\cos^4 x + C.$$

$$13. \int \frac{\sin x dx}{2-3\cos x}; \text{ Ats.: } -\frac{1}{3}\ln|2-3\cos x| + C.$$

$$14. \int \operatorname{ctg} x dx; \text{ Ats.: } \ln|\sin x| + C.$$

$$15. \int \frac{x-2}{x^2-4x+3} dx; \text{ Ats.: } \frac{1}{2}\ln|x^2-4x+3| + C.$$

$$16. \int \frac{3x-2}{x^2+4} dx; \text{ Ats.: } \frac{3}{2}\ln|x^2+4| - \arctg \frac{x}{2} + C.$$

$$17. \int \sqrt{x} \ln x dx; \text{ Ats.: } \frac{2}{3}x\sqrt{x} \left(\ln x - \frac{2}{3} \right) + C.$$

$$18. \int \frac{x^3-2x+1}{(x+2)^2} dx; \text{ Ats.: } \frac{x^2}{2} - 4x + 10\ln|x+2| + \frac{3}{x+2} + C.$$

$$19. \int (x^4 - 5x^2 + 3) dx; \text{ Ats.: } \frac{x^5}{5} - \frac{5x^3}{3} + 3x + C.$$

$$20. \int x(x+1)(x+2) dx; \text{ Ats.: } \frac{x^4}{4} + x^3 + x^2 + C.$$

$$21. \int (1-x^3)^2 dx; \text{ Ats.: } x - \frac{x^4}{2} + \frac{x^7}{7} + C.$$

$$22. \int \frac{dx}{\sqrt[3]{x^2}}; \text{ Ats.: } 3\sqrt[3]{x} + C$$

$$23. \int \frac{1-3x+4x^2}{x} dx; \text{ Ats.: } \ln|x| - 3x + 2x^2 + C.$$

$$24. \int (4e^x + 3\sin x) dx; \text{ Ats.: } 4e^x - 3\cos x + C.$$

$$25. \int \frac{1+\sqrt{1-x^2}}{\sqrt{1-x^2}} dx; \text{ Ats.: } x + \arcsin x + C.$$

$$26. \int (8x+5)^{10} dx; \text{ Ats.: } \frac{1}{88}(8x+5)^{11} + C.$$

$$27. \int \sqrt{6x-5} dx; \text{ Ats.: } \frac{1}{9}\sqrt{(6x-5)^3} + C.$$

$$28. \int \frac{dx}{x \ln x}; \text{ Ats.: } \ln|\ln x| + C.$$

$$29. \int \frac{\sqrt{tg x}}{4\cos^2 x} dx; \text{ Ats.: } \frac{1}{6}\sqrt{tg^3 x} + C.$$

$$30. \int \sqrt{1-\sin x} \cos x dx; \text{ Ats.: } -\frac{2}{3}\sqrt{(1-\sin x)^3} + C.$$

$$31. \int \cos^2 x dx; \text{ Ats.: } \frac{x}{2} + \frac{\sin 2x}{4} + C.$$

$$32. \int \sin^3 x \cos^3 x dx; \text{ Ats.: } \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C.$$

$$33. \int \cos^5 x dx; \text{ Ats.: } \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C.$$

$$34. \int \sin x \sin 5x dx; \text{ Ats.: } \frac{1}{8}\sin 4x - \frac{1}{12}\sin 6x + C.$$

$$35. \int x^2 \ln x dx; \text{ Ats.: } \frac{x^3}{9}(3\ln x - 1) + C.$$

$$36. \int (2x+1)e^{2x} dx; \text{ Ats.: } xe^{2x} + C.$$

$$37. \int (x+2)^{3^x} dx; \text{ Ats.: } \frac{3^x}{\ln^2 3} ((x+2)\ln 3 - 1) + C.$$

$$38. \int \frac{dx}{x^2+3x}; \text{ Ats.: } \frac{1}{3}\ln\left|\frac{x}{x+3}\right| + C.$$

$$39. \int \frac{x dx}{x^2+x-2}; \text{ Ats.: } \ln\sqrt[3]{(x+2)^2|x-1|} + C.$$

$$40. \int \frac{1-8x}{2x^2+x} dx; \text{ Ats.: } 3\ln\left|\frac{x}{2x+1}\right| - 2\ln|2x^2+x| + C.$$

41. $\int (x+2)\cos x dx$; Ats.: $(x+2)\sin x + \cos x + C$.
42. $\int \frac{\ln x}{x^2} dx$; Ats.: $-\frac{1+\ln x}{x} + C$.
43. $\int \sqrt{8-2x} dx$; Ats.: $-\frac{\sqrt{(8-2x)^3}}{3} + C$.
44. $\int \sin^3 x \cos x dx$; Ats.: $\frac{1}{4}\sin^4 x + C$.
45. $\int e^{-x^3} x^2 dx$; Ats.: $-\frac{1}{3}e^{-x^3} + C$.
46. $\int (2x+1)\cos 2x dx$; Ats.: $\left(x + \frac{1}{2}\right)\sin 2x + \frac{1}{2}\cos 2x + C$.
47. $\int x \cos^2 x dx$; Ats.: $\frac{x^2}{4} + \frac{1}{4}x \sin 2x + \frac{1}{8}\cos 2x + C$.
48. $\int \left(x^2 + 2x + \frac{1}{x}\right) dx$; Ats.: $\frac{x^3}{3} + x^2 + \ln|x| + C$.
49. $\int \frac{1+\sqrt[4]{x}}{x+\sqrt{x}} dx$; Ats.: $4\sqrt[4]{x} + 2\ln(\sqrt{x}+1) - 4\operatorname{arctg}\sqrt[4]{x} + C$.
50. $\int \frac{x-2}{x^3} dx$; Ats.: $\frac{1-x}{x^2} + C$.
51. $\int (\sqrt{x} + \sqrt[3]{x}) dx$; Ats.: $x\left(\frac{2}{3}\sqrt{x} + \frac{3}{4}\sqrt[3]{x}\right) + C$.
52. $\int e^{-3x} dx$; Ats.: $-\frac{1}{3}e^{-3x} + C$.
53. $\int \frac{\cos x dx}{\sin^4 x}$; Ats.: $-\frac{1}{3\sin^3 x} + C$.
54. $\int \sin x \cos x dx$; Ats.: $\frac{\sin^2 x}{2} + C$.
55. $\int \sqrt[3]{x^3-8x^2} dx$; Ats.: $\frac{1}{4}\sqrt[3]{(x^3-8)^4} + C$.
56. $\int \frac{x^2 dx}{x^2+1}$; Ats.: $x - \operatorname{arctg} x + C$.
57. $\int \frac{x^4 dx}{x^2-3}$; Ats.: $\frac{x^3}{3} + 3x - \frac{3\sqrt{3}}{2}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + C$.
58. $\int x^2 \cos x dx$; Ats.: $x^2 \sin x + 2x \cos x - 2 \sin x + C$.
59. $\int \sqrt{x} \ln x dx$; Ats.: $\frac{2}{3}\sqrt{x^3}\left(\ln|x| - \frac{2}{3}\right) + C$.
60. $\int (x^4 + 7^x) dx$; Ats.: $\frac{1}{5}x^5 + \frac{7^x}{\ln 7} + C$.
61. $\int \frac{9x^2-1}{3x-1} dx$; Ats.: $\frac{3}{2}x^2 + x + C$.
62. $\int \frac{\cos x dx}{1-2\sin x}$; Ats.: $-\frac{1}{2}\ln|1-2\sin x| + C$.
63. $\int \frac{2e^x dx}{(2+e^x)^2}$; Ats.: $-\frac{2}{2+e^x} + C$.
64. $\int \frac{dx}{9x^2+16}$; Ats.: $\frac{1}{12}\operatorname{arctg}\frac{3x}{4} + C$.
65. $\int \frac{dx}{\cos^2 2x}$; Ats.: $\frac{1}{2}\operatorname{tg} 2x + C$.
66. $\int \frac{dx}{\sqrt{1-(x+2)^2}}$; Ats.: $\arcsin(x+2) + C$.
67. $\int \frac{x^2 dx}{x^6+4}$; Ats.: $\frac{1}{6}\operatorname{arctg}\frac{x^3}{2} + C$.
68. $\int \frac{dx}{x^2+8x-9}$; Ats.: $\frac{1}{10}\ln\left|\frac{x-1}{x+9}\right| + C$.
69. $\int \frac{2x+3}{(x-2)(x+5)} dx$; Ats.: $\ln|x-2| + \ln|x+5| + C$.
70. $\int (x-1)e^{2x} dx$; Ats.: $\frac{1}{2}(x-1)e^{2x} - \frac{1}{4}e^{2x} + C$.
71. $\int \frac{(x^3+1)dx}{x-1}$; Ats.: $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x-1| + C$.
72. $\int \sin x \cos 2x dx$; Ats.: $\frac{1}{2}\left(\cos x - \frac{1}{3}\cos 3x\right) + C$.
73. $\int x\sqrt{x-1} dx$; Ats.: $\frac{2}{15}(3x^2-x-2)\sqrt{x-1} + C$.
74. $\int \frac{\sqrt{x} dx}{x+1}$; Ats.: $2\sqrt{x} - 2\operatorname{arctg}\sqrt{x} + C$.
75. $\int \frac{1-2\sqrt{x}}{1+2\sqrt{x}} dx$; Ats.: $-x + 2\sqrt{x} - \ln(1+2\sqrt{x}) + C$.
76. $\int \frac{(1+2x^2)dx}{x^2(1+x^2)}$; Ats.: $\operatorname{arctg} x - \frac{1}{x} + C$.
77. $\int 2\sin^2 \frac{x}{2} dx$; Ats.: $x - \sin x + C$.
78. $\int \frac{dx}{(2x-3)^5}$; Ats.: $-\frac{1}{8(2x-3)^4} + C$.
79. $\int 2x\sqrt{x^2+1} dx$; Ats.: $\frac{2}{3}\sqrt{(x^2+1)^3} + C$.
80. $\int \frac{xdx}{\sqrt{x^2+1}}$; Ats.: $\sqrt{x^2+1} + C$.
81. $\int \frac{\sqrt{\ln x}}{x} dx$; Ats.: $\frac{2}{3}\sqrt{(\ln x)^3} + C$.
82. $\int \frac{3+x}{3-x} dx$; Ats.: $-x - 6\ln|3-x| + C$.

8. Apibrėžtinis integralas

1. $\int_{-3}^1 (2x+5)dx$; Ats.: 12.
2. $\int_{-2}^1 (3x^2 - 5x - 2)dx$; Ats.: 10,5.
3. $\int_1^4 \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$; Ats.: $2\frac{2}{3}$.
4. $\int_1^9 \frac{x+1}{\sqrt{x}} dx$; Ats.: $21\frac{1}{3}$.
5. $\int_{\frac{p}{4}}^{\frac{p}{4}} \cos x dx$; Ats.: $\sqrt{2}$.
6. $\int_0^{\sqrt{3}} \frac{2dx}{1+x^2}$; Ats.: $\frac{2}{3}p$.
7. $\int_{\frac{\sqrt{3}}{2}}^1 \frac{2dx}{\sqrt{1-x^2}}$; Ats.: $\frac{p}{3}$.
8. $\int_{-1}^2 (x^2 + 2x + 1)dx$; Ats.: 9.
9. $\int_{-1}^0 (x^3 + 2x)dx$; Ats.: $-1\frac{1}{4}$.
10. $\int_1^8 \sqrt[3]{x^2} dx$; Ats.: $18\frac{3}{5}$.
11. $\int_8^{27} \frac{dx}{\sqrt[3]{x}}$; Ats.: 7,5.
12. $\int_{-1}^1 e^x dx$; Ats.: $\frac{e^2 - 1}{e}$.
13. $\int_0^1 e^{3x} dx$; Ats.: $\frac{e^3 - 1}{3}$.
14. $\int_1^e \frac{dx}{x}$; Ats.: 1.
15. $\int_0^1 \frac{dx}{x+2}$; Ats.: $\ln 1,5$.
16. $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$; Ats.: $\frac{p}{2}$.

9. Apibrėžtinio integralo skaičiavimas keičiant kintamąjį

1. $\int_4^5 (4-x)^3 dx$; Ats.: $-\frac{1}{4}$.
2. $\int_0^1 \frac{dx}{(3x+1)^4}$; Ats.: $\frac{7}{64}$.
3. $\int_0^3 \sqrt[3]{3x-1} dx$; Ats.: $3\frac{3}{4}$.
4. $\int_1^5 \sqrt{(2x-1)^3} dx$; Ats.: 12.
5. $\int_0^5 \sqrt{3x+1} dx$; Ats.: 3.
6. $\int_0^1 (2x^3 + 1)^4 x^2 dx$; Ats.: $\frac{1}{15}$.
7. $\int_{-1}^2 (x^2 - 1)^3 x dx$; Ats.: $10\frac{1}{8}$.
8. $\int_0^2 \frac{4x dx}{(x^2 - 1)^3}$; Ats.: $\frac{8}{9}$.
9. $\int_0^2 9\sqrt{x^3 + 1} x^2 dx$; Ats.: 52.
10. $\int_{\sqrt{5}}^{2\sqrt{2}} \frac{x dx}{\sqrt{3x^2 + 1}}$; Ats.: $\frac{1}{3}$.
11. $\int_1^{\sqrt{3}} \frac{32x dx}{(x^2 + 1)^5}$; Ats.: $\frac{15}{64}$.
12. $\int_0^{\frac{p}{2}} \sqrt{3 \sin x + 1} \cos x dx$; Ats.: $1\frac{5}{9}$.
13. $\int_{\frac{3p}{2}}^{2p} \sqrt{1 - \cos x} \sin x dx$; Ats.: $-\frac{2}{3}$.
14. $\int_0^{\frac{p}{3}} \frac{\sin x dx}{3 - \cos x}$; Ats.: $\ln 1,25$.
15. $\int_0^{\frac{p}{2}} \frac{\cos x dx}{2 + \sin x}$; Ats.: $\ln 1,5$.
16. $\int_0^{\frac{p}{6}} e^{\sin x} \cos x dx$; Ats.: $\sqrt{e} - 1$.

$$17. \int_0^{\frac{1}{2}} e^{-2x} dx; \quad \text{Ats.: } \frac{1-e}{2}.$$

$$18. \int_{\frac{p}{12}}^{\frac{p}{8}} \sin 2x dx; \quad \text{Ats.: } \frac{\sqrt{3}-\sqrt{2}}{4}.$$

$$19. \int_{\frac{p}{8}}^{\frac{p}{6}} \frac{dx}{\cos^2 2x}; \quad \text{Ats.: } \frac{\sqrt{3}-1}{2}.$$

$$20. \int_2^4 \frac{dx}{\sqrt{16-x^2}}; \quad \text{Ats.: } \frac{p}{3}.$$

$$21. \int \frac{4x dx}{(x^2-1)^3}; \quad \text{Ats.: } \frac{8}{9}.$$

$$22. \int_0^{\sqrt{5}} \sqrt{(4-3x)^3} dx; \quad \text{Ats.: } -132 \frac{4}{15}.$$

$$23. \int_0^{\sqrt{5}} \frac{3x dx}{\sqrt{x^2+4}}; \quad \text{Ats.: } 3.$$

$$24. \int_0^2 \frac{2x dx}{\sqrt{2x^2+1}}; \quad \text{Ats.: } 2.$$

$$25. \int_0^{\frac{p}{4}} \sin 4x dx; \quad \text{Ats.: } 0,5.$$

$$26. \int_{\frac{p}{6}}^{\frac{p}{2}} \frac{3 \cos x dx}{2 \sin x + 1}; \quad \text{Ats.: } 1,5 \ln 1,5.$$

$$27. \int_{\frac{\sqrt{3}}{2}}^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}; \quad \text{Ats.: } \frac{p}{2}.$$

$$28. \int_0^{\sqrt{3}} \frac{dx}{9+x^2}; \quad \text{Ats.: } \frac{p}{18}.$$

10. Apibrėžtinio integralo integravimas dalimis

$$1. \int_0^1 x e^{2x} dx; \quad \text{Ats.: } \frac{1}{4}(e^2+1)$$

$$2. \int_1^2 \ln x dx; \quad \text{Ats.: } \ln 4 - 1.$$

$$3. \int_1^2 x \ln x dx; \quad \text{Ats.: } \ln 4 - \frac{3}{4}.$$

$$4. \int_0^p x \cos x dx; \quad \text{Ats.: } -2.$$

$$5. \int_0^{\frac{p}{2}} x \sin x dx; \quad \text{Ats.: } 1.$$

$$6. \int_e^4 x \ln x; \quad \text{Ats.: } 8 \ln 4 - 4 - \frac{e^2}{4}.$$

$$7. \int_0^1 \arcsin x dx; \quad \text{Ats.: } \frac{p}{2} - 1.$$

$$8. \int_0^1 x \arctg x dx; \quad \text{Ats.: } \frac{p}{4} - \frac{1}{2}.$$

$$9. \int \ln^2 x dx; \quad \text{Ats.: } e - 2.$$

$$10. \int_0^1 x e^{-x} dx; \quad \text{Ats.: } 1 - \frac{2}{e}.$$

$$11. \int_0^{\frac{p}{4}} x \sin 2x dx; \quad \text{Ats.: } \frac{1}{4}.$$

$$12. \int_1^2 x^2 \ln x dx; \quad \text{Ats.: } \frac{8}{3} \ln 2 - \frac{7}{9}.$$

$$13. \int_0^{\frac{1}{2}} \arccos x dx; \quad \text{Ats.: } 1 - \frac{p}{6} - \frac{\sqrt{3}}{2}.$$

$$14. \int_0^{2p} x^2 \cos x dx; \quad \text{Ats.: } 4p.$$

11. Netiesioginiai integralai

$$1. \int_1^{+\infty} \frac{dx}{x}; \quad \text{Ats.: diverguoja.}$$

$$2. \int_1^{+\infty} \frac{dx}{1+x^2}; \quad \text{Ats.: } \frac{p}{4}.$$

$$3. \int_{-\infty}^0 \frac{dx}{x+1}; \quad \text{Ats.: diverguoja.}$$

$$4. \int_{-\infty}^{+\infty} \frac{dx}{x^2+4x+9}; \quad \text{Ats.: } \frac{p\sqrt{5}}{5}.$$

$$5. \int_0^{+\infty} x e^{-x^2} dx; \quad \text{Ats.: } 0,5.$$

$$6. \int_0^{+\infty} x e^{-x} dx; \quad \text{Ats.: } 1.$$

$$7. \int_e^{+\infty} \frac{dx}{x \ln^2 x}; \quad \text{Ats.: } 1.$$

$$8. \int_{-\infty}^0 \frac{x+1}{x^2+1} dx; \quad \text{Ats.: diverguoja.}$$

$$9. \int_0^{+\infty} x \cos 2x dx; \quad \text{Ats.: diverguoja.} \quad 10. \int_1^{+\infty} \frac{dx}{x^2(1+x)}; \quad \text{Ats.: } 1 - \ln 2.$$

12. Apibrėžtinio integralo taikymai

Plokščios figūros plotas

Rasti figūrų, apribotų linijomis, plotus:

$$1. 2x+3y=9, y=0, x=1, x=4; \quad \text{Ats.: } 10.$$

$$3. 4x+5y=20, x=0, y=0; \quad \text{Ats.: } 10.$$

$$5. y=3x^2+3, y=0, x=-2, x=1; \quad \text{Ats.: } 18.$$

$$7. y = -\frac{1}{3}x^2 + 3, \quad y = 0, \quad x = -2, x = 3; \quad \text{Ats.: } 11\frac{1}{9}.$$

$$9. y=-x^2+8x+4, y=0; \quad \text{Ats.: } 120.$$

$$11. y=-x^2+6, x+y=4; \quad \text{Ats.: } 4,5.$$

$$13. y = \frac{4}{x^2}, y=7-3x; \quad \text{Ats.: } 0,5.$$

$$15. y=x^2-2x+2, y=2+4x-x^2; \quad \text{Ats.: } 9.$$

$$17. y=2x^2-4x+3, x+2y=6; \quad \text{Ats.: } 1\frac{17}{64}.$$

$$19. xy=4, x=1, x=4, y=0; \quad \text{Ats.: } 8\ln 2.$$

$$2. y=3x^5-1, y=0, x=2, x=4; \quad \text{Ats.: } 16.$$

$$4. y=x^2+1, y=0, x=0, x=2; \quad \text{Ats.: } 4\frac{2}{3}.$$

$$6. y=-x^2+9, y=0; \quad \text{Ats.: } 36.$$

$$8. y=-x^2+6x-5, y=0, x=2, x=3; \quad \text{Ats.: } 3\frac{2}{3}.$$

$$10. y=-x^2+5x-4, y=0; \quad \text{Ats.: } 4,5.$$

$$12. y=x^2, x-2y=-6; \quad \text{Ats.: } 7\frac{7}{48}.$$

$$14. y=x^2, y=2x-x^2; \quad \text{Ats.: } 1/3.$$

$$16. y=x^2-2x+2, 2x+y=6; \quad \text{Ats.: } 10\frac{2}{3}.$$

$$18. y=4-x^2, y=0; \quad \text{Ats.: } \frac{32}{3}.$$

$$20. y=\ln x, x=e, y=0; \quad \text{Ats.: } 1.$$

Sukinio tūris

1. Apskaičiuokite tūrius sukinių, kurie gaunami duotų kreivių apribotas figūras sukant apie OX ašį:

$$1. y^2=2x, x=3, y=0; \quad \text{Ats.: } 9p.$$

$$2. y^2=6x, x=5, y=0; \quad \text{Ats.: } 75p.$$

$$4. y=5\sqrt{x}, x=1, x=4, y=0; \quad \text{Ats.: } 7,5p.$$

$$5. y=3x, y=0, x=2; \quad \text{Ats.: } 24p.$$

$$6. y=2x-x^2, y=0; \quad \text{Ats.: } \frac{16}{15}p.$$

$$7. y=5x^3, y=0, x=2; \quad \text{Ats.: } \frac{16}{3}p.$$

$$8. xy=4, x=1, x=4, y=0; \quad \text{Ats.: } 12p.$$

$$9. y=5x^3, y=\sqrt{x}; \quad \text{Ats.: } \frac{5}{14}p.$$

$$10. y=(x-1)^2, y=1; \quad \text{Ats.: } \frac{8}{5}p.$$

$$11. y=\sqrt{4-x^2}, x-y+2=0, y=0; \quad \text{Ats.: } 8p.$$

$$12. y=x, y=\frac{1}{x}, x=3; \quad \text{Ats.: } 8p.$$

$$13. y=\frac{x^2}{4}, y=\frac{x^3}{8}, \quad \text{Ats.: } \frac{4}{35}p.$$

$$14. y=\sin x, y=\frac{2}{p}x, \quad 0 \leq x \leq \frac{p}{2}; \quad \text{Ats.: } \frac{p^2}{12}.$$

$$15. y=\sin x, y=\cos x, y=0, \quad 0 \leq x \leq \frac{p}{2}; \quad \text{Ats.: } \frac{p(p-2)}{4}.$$

2. Apskaičiuokite tūrius sukinių, kurie gaunami duotų kreivių apribotas figūras sukant apie OY ašį:

$$16. y=2x-x^2, y=0; \quad \text{Ats.: } \frac{8}{3}p.$$

$$17. y=x\sqrt{x}, x=4, y=0; \quad \text{Ats.: } \frac{512}{7}p.$$

$$18. y=x^3, x=0, y=8; \quad \text{Ats.: } 19,2p.$$

$$19. x^2-y^2=4, y=-2, y=2; \quad \text{Ats.: } \frac{64}{3}p.$$

$$20. y=\ln x, x=2, y=0; \quad \text{Ats.: } 2p\left(2\ln 2 - \frac{3}{4}\right).$$